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Individual Differences in Information-Processing Rate and Amount: Implications for Group Differences in Response Latency

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Research on group differences in response latency often has as its goal the detection of Group × Treatment interactions. However, accumulating evidence suggests that response latencies for different groups are often linearly related, leading to an increased likelihood of finding spurious overadditive interactions in which the slower group produces a larger treatment effect. The authors propose a rate–amount model that predicts linear relationships between individuals and that includes global processing parameters based on large-scale group differences in information processing. These global processing parameters may be used to linearly transform response latencies from different individuals to a common information-processing scale so that small-scale group differences in information processing may be isolated. The authors recommend linear regression and z-score transformations that may be used to augment traditional analyses of raw response latencies.

Measurement of response latency to perform simple judgments has played an important role in research attempting to identify and isolate fundamental mental operations (Donders, 1868/1969; Luce, 1986; Posner, 1978; Sternberg, 1969; Townsend & Ashby, 1983; Townsend & Schweickert, 1989). Experimental tasks within this tradition are typically performed quickly (i.e., in less than 2 s) and accurately (i.e., above 90% accuracy) by the average participant. The fundamental assumption underlying this scientific endeavor is that differences in response latencies across experimental conditions reflect differences in the amount of time taken to perform various fundamental mental operations under various experimental conditions. Given this view, it seems natural to propose that physical time is the appropriate scale of measurement to probe the mental architecture. Townsend (1992), for example, has argued that when used appropriately, response latency "can be employed in a way that is nothing more nor less than the application of physical time to psychological science" (p. 107). The assumption of a physical scale of measurement does not, however, guarantee that equivalent psychological effects for different individuals will necessarily lead to equivalent effects in response latencies, measured in physical units such as milliseconds, for these same individuals.

Cognitive Speed

To see how this might be the case, consider two hypothetical individuals, with the first individual taking twice as long as the second individual, minus a constant of 300 ms, to produce a response across a wide range of cognitive tasks and/or experimental conditions (see Cerella, 1990, 1991; Myerson, Ferraro, Hale, & Lima, 1992; Salthouse, 1985, for similar arguments). That is, the average response latency for the slower of these two hypothetical individuals will be two times that of the faster individual minus 300 ms for any given experimental condition. Assume further that these two individuals have just completed a semantic priming experiment, with the faster individual producing average response latencies of, say, 400 ms and 500 ms in the related and unrelated conditions, respectively (i.e., a 100-ms semantic priming effect). Figure 1 presents a plot of the results of our hypothetical priming experiment. As can be seen, there is an overadditive interaction such that the slower individual has produced a larger priming effect. Because the slope of the linear function relating the response latencies of the slower individual to those of the faster individual is 2, the effect size for the slower individual will generally be twice that of the faster individual (i.e., a 200-ms semantic priming effect).

If the results of the hypothetical semantic priming study described above were to be viewed in isolation, and if response

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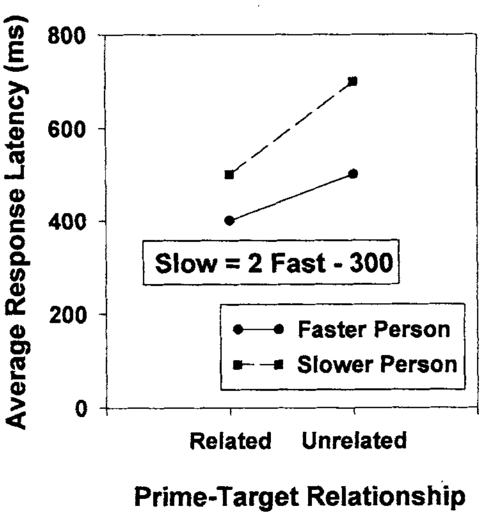


Figure 1. Interaction plot of mean latencies for two individuals whose performance is constrained by a constant linear relationship across experimental conditions.

latency were to be taken as a physical scale of measure of fundamental mental operations common to both individuals, then one might conclude that the first individual has exhibited twice as much semantic priming as the second individual. However, this view does not take the general systematic relationship between the individuals into account. Given that the slower individual will produce effects that are twice as large as those for the faster individual across a wide range of tasks, a more general account would be that the two individuals show semantic priming effects that are psychologically equivalent. The true difference between the two hypothetical individuals is one of overall information-processing rate, what we term *cognitive speed.* As described later, the slope of the linear relationship between the response latencies of the slower and faster individuals can be viewed as an indication of their relative cognitive speeds (i.e., the slope of 2 indicates that the slower individual processes information at twice the rate of the faster individual). Of course, in any real-world situation, individuals might differ both in terms of overall processing speed and in terms of some specific component operation. Our approach suggests that specific differences will remain in response latencies appropriately rescaled to correct for general differences.

Amount of Information Processing

The intercept of the linear relationship between the response latencies of the two hypothetical individuals is determined by the amount of information processing performed that is constant across various experimental conditions for each of the individuals in our hypothetical example. We must defer discussion of why this is the case until after presentation of an individual-differences model that includes both processing speed and amount parameters. For now, consider the idea that there are significant individual differences in the amount of information processing performed in making a particular decision to respond (e.g., pressing one of two potential response buttons) in a typical cognitive task. Indeed, two individuals with the same cognitive speed may reliably produce different response latencies because they differ in the amount of information processing performed to reach a point where a decision can be made to choose and initiate a particular response.

Group Differences in Response Latency

If we expand the arguments above to two different groups of interest, the simple example above illustrates three important points regarding the interpretation of studies using response latency as a dependent measure. First, Group × Treatment interactions are not easily interpreted in the face of differences in overall latency (e.g., Cerella, 1990, 1991; Salthouse, 1985). Second, knowledge of a systematic relationship between latencies produced by different individuals can provide the basis for a correction of latencies for individual differences in information-processing rate and amount. As discussed later, correction for individual differences in information-processing rate and amount will help reduce false positives in studies designed to detect specific group differences in fundamental mental operations. The model we propose predicts that response latency scales for various individuals are general linear transformations of each other in much the same way that the Fahrenheit and Centigrade scales of temperature are linear transformations of each other requiring both an additive and a multiplicative constant. Third, it may be the case that group differences in information-processing rate and amount will be as interesting as more specific group differences in a particular mental operation.

Overview

We begin by reviewing evidence for a systematic (generally linear) slowing of response latencies across various groups of interest. We then consider a general framework for understanding the large-scale structure of response latencies across a wide range of tasks and individuals in terms of the rate and amount of cognitive processing performed. Assuming that information processing in the human nervous system is based directly on physical processes, we can define the same relationship between time, rate, and amount that must hold for any physical process. That is, time to complete a task (i.e., response latency) is equal to the ratio of the amount of processing performed and the rate (i.e., cognitive speed). The resultant multiplicative rate–amount model of response latency imposes a specific structure on the expected value of response latencies and can act as a theoretical bridge to guide our choice of method to transform response latencies to a scale that is equivalent across individuals with respect to overall cognitive speed and processing amount.

In this article, we discuss the potential methods for obtaining estimates of cognitive speed and processing amount and the theoretical importance of documenting and understanding large-scale group differences in information-processing rate and amount across a wide range of tasks. These procedures allow for the identification of more global group differences than does the more traditional focus on fundamental mental operations. For example, the relationship between older adult latencies and younger adult latencies has been shown to differ within broad classes of processing, such as lexical versus nonlexical tasks (e.g., Hale, Lima, & Myerson, 1991; Lima, Hale, & Myerson, 1991), or with differential involvement of working memory (e.g., Mayr & Kliegl, 1993).

Although we use examples taken from the cognitive aging literature to motivate our discussion, there is increasing evidence that the response latencies of many diverse groups are systematically related. Therefore, the discussion that follows should be of general interest to those researchers (e.g., developmental psychologists, gerontologists, and cognitive neuropsychologists) interested in examining group differences in response latencies. Furthermore, the present article should also be of interest to those interested in individual and group differences in information-processing rate and amount. We have chosen the cognitive aging literature both because of our familiarity with it and because this literature has been struggling with issues regarding group differences in response latency to perform cognitive tasks (e.g., Cerella, 1985).

Finally, even though a primary emphasis is on a theoretically based approach to transforming response latencies to control for individual differences in cognitive speed and processing amount, it should be noted that we are most certainly not advocating an extreme view that would preclude investigators from analyzing raw response latencies in a traditional manner. In fact, we encourage comparison of similar analyses performed on raw and transformed response latencies. Such comparisons can provide important information regarding the most appropriate interpretation of observed group differences in response latency.

Systematic Group Differences in Response Latency: Brinley Plot Evidence

A number of studies have documented the fact that older adults' response latencies are systematically slower than younger adult latencies across a wide range of tasks (e.g., Cerella, 1985; Cerella, Poon, & Williams, 1980; Hale et al., 1991; Lima et al., 1991; Salthouse, 1985; Smith, Poon, Hale, & Myerson, 1988; see Bashore, 1994, for a recent review). Table 1 lists several meta-analytic studies demonstrating strong linear relationships between response latencies for younger and older adults.

Lima et al. (1991) obtained a sample of 19 studies of lexical processing in younger and older adults appearing in 11 journals during the years 1975 to 1987 and 7 additional studies on nonlexical processing appearing in the *Journal of Gerontology* during the years 1975 to 1984. Figure 2 is a scatter plot (often called Brinley plots, after Brinley, 1965) of older adult group means as a function of younger adult group means in the same experimental conditions, adapted from the results reported by Lima et al.

An examination of Figure 2 confirms a strong linear relationship between younger and older adult response latency that is compelling both to the naked eye and in terms of the proportion of overall variance accounted for ($r^2 = .91$). It is worth noting that Figure 2 is based on 26 different studies (and presumably 26 different samples of younger and older adults). Brinley plots of the data generated by the same sample of older and younger adults participating in all experimental conditions often provide linear fits that account for 98% or more of the variance (e.g., Balota & Ferraro, 1992; Brinley, 1965; Hale et al., 1991; Madden, Pierce, & Allen, 1993; Maylor & Rabbitt, 1994; Mayr & Kliegl, 1993; Salthouse & Somberg, 1982). In fact, the pattern of slowing in normal aging has been found to be systematic enough to provide good fits to well-specified quantitative models (e.g., Cerella, 1985; Myerson, Hale, Wagstaff, Poon, & Smith, 1990; see Cerella, 1990, and Cerella & Hale, 1994, for reviews).

Evidence for a systematic general slowing factor among an increasingly wide range of populations of interest is accumulating in the literature. Studies of normal aging (e.g., Cerella et al., 1980; Hale et al., 1991), closed-head injured patients (e.g., Ferraro, 1996), individuals with Alzheimer's disease (e.g., Nebes & Brady, 1992; Nebes & Madden, 1988), individuals under the influence of

Table 1
Parameter Estimates and Proportion of Accounted Variance for Linear Brinley Functions for Conditions in Which Younger Adult Latencies Are Less Than 2 s

Study	No. of experiments[a]	No. of conditions	Formula[b]	r^2
Meta-analytic				
Cerella (1985)[c]				
Lexical and nonlexical tasks	14	94	O = 1.54Y − 150[d]	.96
Lima et al. (1991)				
All tasks				
Lexical decision tasks	10	90	O = 1.48Y − 68	.95
Other lexical tasks	11	76	O = 1.47Y − 101	.97
Nonlexical tasks	8	59	O = 2.05Y − 385	.92
Nebes & Brady (1992)				
Lexical and nonlexical tasks	10	61	O = 1.37Y − 77	.90
Empirical				
Hale et al. (1991)				
Four nonlexical tasks	1	11	O = 2.15Y − 413	.99
Hale et al. (1995)				
Seven nonlexical tasks	1	22	O = 2.74Y − 704	.98
Madden et al. (1993)				
Lexical decision	4	106	O = 1.58Y − 183	.91

Note. O = older adult; Y = younger adult.
[a] An experiment indicates a specific sample of participants and may contain more than one task. [b] Functions are expressed in millisecond units. [c] Two conditions with younger adult latencies between 2 and 3 s included. [d] Function estimated from Cerella (1990).

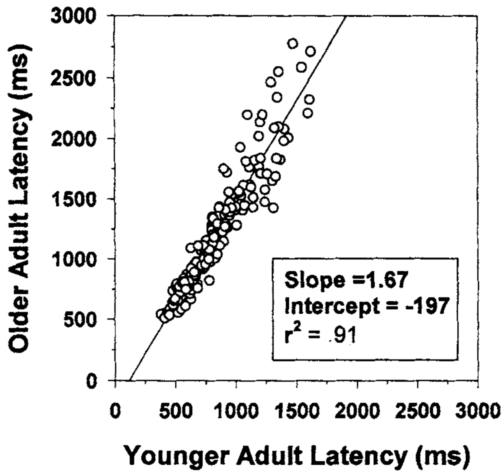


Figure 2. Scatter plot of older adult mean latency as a function of younger adult mean latency from the same experimental conditions. Data are from "How General Is General Slowing? Evidence From the Lexical Domain," by S. D. Lima, S. Hale, and J. Myerson, 1991, *Psychology and Aging, 6,* pp. 416–425. Copyright 1991 by the American Psychological Association. Reprinted with permission.

alcohol (e.g., Maylor, Rabbitt, James, & Kerr, 1992), individuals differing in intelligence (e.g., Vernon & Jensen, 1984), and healthy children (e.g., Hale, Fry, & Jessie, 1993; Kail, 1991, 1992) suggest that groups of interest often produce response latencies that are generally slowed and systematically related to latencies from some reference group.

However, general slowing is not sufficient to explain the full range of age-related differences in response latencies reported in the literature (see Fisk & Fisher, 1994, for a detailed discussion of this issue); some component processes may be affected more by age than others (e.g., Balota, Black, & Cheney, 1992; Balota & Ferraro, 1993, 1996; Cerella, 1985; Duchek, Balota, Faust, & Ferraro, 1995; Fisk, Fisher, & Rogers, 1992; Hale, Myerson, & Wagstaff, 1987; Lima et al., 1991; Madden et al., 1993; Maylor & Rabbitt, 1994; Mayr & Kliegl, 1993; Spieler, Balota, & Faust, 1996). Thus, the question of how to detect deviations from general slowing has become increasingly important (e.g., Cerella, 1991; Fisher & Glaser, 1996; Fisk et al., 1992; Madden, Pierce, & Allen, 1992; Myerson, Wagstaff, & Hale, 1994; Salthouse, 1992). One general approach suggested by a number of researchers is to use statistical inference tests in which the null hypothesis is shifted away from the assumption of null group differences and toward the assumption of general slowing (e.g., Burke, White, & Diaz, 1987; Kliegl & Mayr, 1992; Madden et al., 1993; Salthouse, 1992). However, analyzing deviations from a general slowing function in terms of raw (untransformed) units of time is problematic in that the same issues regarding the effects of individual differences in information-processing rate and amount that apply to raw response latencies will also apply to raw deviation scores. In the present article, we propose a psychometric model of response latency that predicts the appropriate general slowing function and we examine several candidate transformations of raw response latency designed to control for individual differences in overall level of response. This will have the logical effect of removing group differences in cognitive speed and processing amount and is therefore logically equivalent to a null hypothesis that assumes systematic general slowing between groups. However, deviations from the general slowing function, now in appropriately transformed units, will remain. Before presenting the details of such transformations, we first present our proposed psychometric model of response latency.

Information-Processing Rate and Amount

Processing models that include the product of rate and amount of information processing have found widespread usage in psychology. Hick (1952) assumed that the amount of information processing required for an optimal decision maker in a relatively simple forced-choice task was completely determined by the information content of the stimulus set (i.e., as specified by information theory [Shannon, 1948], the number of possible alternatives in combination with their probability of occurrence). He proposed a model of average choice response latency that was a product of a rate parameter and the amount of information processing required:

$$L = a \log_2 (N + 1), \tag{1}$$

where L is the average response latency, a is a rate parameter, and N is the number of possible response alternatives. A number of researchers have explored variations on this theme (Luce, 1986, chap. 10).

Rate-by-amount models have also been proposed for solution times to individual items on tests of ability in the psychometric literature (e.g., Frearson, Eysenck, & Barrett, 1990; Furneaux, 1961; Rasch, 1980). More recently, multiplicative rate–amount models have proven quite successful in explaining average response latencies in specific tasks, such as mental rotation (e.g., Cooper & Shepard, 1973; Shepard & Metzler, 1971), short-term memory scanning (e.g., Sternberg, 1975), and visual search (e.g., Atkinson, Holmgren, & Juola, 1969). Within certain limits, response latencies generated by each of these paradigms can be described nicely by a rate parameter attributed to an individual and an amount parameter (e.g., degrees of mental rotation and number of items to be searched) attributed to each experimental condition. Many recent connectionist models that attempt to fit response latency data make similar rate-by-amount assumptions. Here, information is typically processed in cycles in neural-like units, with more difficult conditions taking more cycles (e.g., Cohen, Dunbar, & McClelland, 1990; Kawamoto, 1993). Because rate of cycles multiplied by time to complete the task will equal the number of cycles necessary to complete the task, predicted response latencies can then be derived from the ratio of the number of cycles and cycles per second.

Random-walk models are another important class of response latency models that are generally consistent with a rate–amount framework (e.g., Luce, 1986, chap. 8). In general, random-walk models predict that responses will occur when information accumulates beyond some threshold and that the accumulation of information over time can be modeled as a random-walk process. Random-walk models assume separate rate and amount parameters (e.g., Luce, 1986, chap. 8; Nosofsky & Palmeri, 1997; Ratcliff, 1978, 1988; Townsend & Ashby, 1983) and are therefore consistent with a latent variable rate–amount framework.

Large- and Small-Scale Structure in Response Latency

The models discussed above all use the ratio of information-processing amount over rate of processing to predict response latencies generated under specific experimental conditions. Thus, there is a general multiplicative principle that runs through all of the rate-by-amount models listed above (and many others not cited) that could potentially be used to better understand the large-scale structure of response latency data and as a basis for correcting for group and individual differences in overall response latency. This general rate-by-amount principle describes the relationship between time and the ratio of amount of processing performed over the rate of processing. Our rate–amount model is based on the following assumptions: (a) that individuals possess a characteristic base rate of information processing (i.e., a cognitive speed) and (b) that experimental conditions require a certain amount of information to be processed (i.e., they have a difficulty) before the average individual can reliably choose and initiate a correct response.

Of course, we are not claiming that information-processing rate or amount will not vary systematically with variation of individual, task, experimental condition, and temporal or psychological context. It is our view, however, that individual differences in overall information-processing rate will interact multiplicatively with the difficulty level of an experimental condition to produce a systematic large-scale structure to response latencies across a wide range (approximately 200 ms to 2,000 ms for the typical undergraduate) of the response latency scale. By contrast, minor fluctuations in information-processing rate and amount due to either manipulation of experimental variables or transient fluctuations in psychological state will tend to produce smaller scale deviations from the overall pattern produced by individual differences.

Individual Differences in Cognitive Speed: Factor Analytic Evidence

Although there is an extensive literature identifying elementary cognitive operations in groups of younger adults within the chronometric information-processing tradition (Posner, 1978), the literature regarding individual differences in elementary cognitive operations is much more limited (see Kyllonen, 1993, for an example of response latencies used in conjunction with more traditional tests of mental abilities within an information-processing framework). However, there is a growing body of work examining the factor structure (using techniques of factor analysis) of response latencies generated by cognitive tasks (e.g., Hunt, Davidson, & Lansman, 1981; Levine, Preddy, & Thorndike, 1987; Vernon, 1983; Vernon & Jensen, 1984; Vernon & Kantor, 1986; Vernon, Nador, & Kantor, 1985).

As discussed above, the existence of a General Speed factor extracted from a covariance matrix using factor analytic techniques is consistent with the idea that individuals possess a global information processing rate, or cognitive speed, that remains relatively stable across variations in test conditions. Table 2 outlines a subset of the evidence for a General Speed factor from a sample of studies that have reported the factor structure of response latencies. These studies differ somewhat in methodology, but all used either principal-components or principal-factors analysis. Although it is tempting to view the striking similarity among these studies as evidence for a General Speed factor, the evidence for such a factor is still preliminary. The studies listed in Table 2 did not choose cognitive tasks to reflect the full range of speeded tasks as they appear in the information-processing literature. Therefore, the first principal component or factor is, at best, only an approximation to a General Speed factor (Humphreys, 1989). Alternatively, the first component or factor may be interpreted as a circumscribed information-processing factor. For instance, Hunt et al. (1981) used seven measures from tasks that all shared the requirement of retrieval of semantic information from long-term memory. They therefore interpreted the strong, single principal component in their results as speed to access long-term memory.

Despite these limitations, it is still interesting to note in Table 2 that (a) mean response latencies are, typically, moderately to very highly positively correlated with each other; (b) principal-components or principal-factors analysis usually produces a single, or at least a first, unrotated factor that accounts for two thirds to three fourths of the overall variance in mean response latency; and

Table 2
Mean and Standard Deviation of Product–Moment Correlations and Factor Loadings Among Selected Studies of Response Latency on Cognitive Tasks

		r				Loading[d]	
Study	No. of tasks[a]	*M*	*SD*	No. of factors[b]	% variance[c]	*M*	*SD*
Hunt et al. (1981)[e]	4	.71	.13	1	75	.86	.08
Vernon (1983)	6	.59	.18	1	66	.77	.16
Vernon & Jensen (1984)	6	.59	.18	1	66	.77	.17
Vernon et al. (1985)	8	.68	.13	1	71	.83	.08
Vernon & Kantor (1986)	8	.73	.10	1	76	.85	.07
Levine et al. (1987)[e]	6	.56	.11	1	64	.80	.06
Kranzler & Jensen (1991)[e]	6	.61	.08	1	68	.82	.03
Gernsbacher & Faust (1991)	3	.72	.09	2	74	.86	.03

[a] Number of distinct tasks included; some tasks included more than one condition. [b] Number of principal components or factors with eigenvalues greater than 1. [c] Percentage of variance in data accounted for by the first unrotated factor or component. [d] Mean and standard deviation of loadings for first unrotated factor or component. [e] Factor structure computed by Mark E. Faust using principal-components analysis of published product–moment correlations.

(c) individual tasks or conditions in these studies load uniformly and highly on the first factor. In fact, Jensen (1988) stated that "in several multivariate studies [of response latency]. . .that I have seen, however, one feature is quite clear: There is always a large General Speed factor along with other relatively smaller factors associated with particular processes" (p. 120).

Several studies have reported correlations between measures of speed of information processing and psychometric tests of intellectual ability. These results are consistent with a General Speed factor that plays an important part in efficient information processing (e.g., Jensen, 1993; Kranzler & Jensen, 1991; Levine et al., 1987; Vernon, 1983; Vernon & Jensen, 1984; Vernon & Kantor, 1986; Vernon, Nador, & Kantor, 1985). Measures of reaction time in cognitive tasks have been found to be correlated with the *g* factor common to traditional psychometric tests of intellectual ability. When the variability common to *g* and response latency is partialed out, the correlation between response latency and scores on psychometric tests approaches zero (Jensen, 1993).

Several studies have found the General Speed factor common to information-processing tasks to be significantly heritable (Baker, Vernon, & Ho, 1991; McGue, Bouchard, Lykken, & Feuer, 1984; Vernon, 1989). McGue et al. tested monozygotic (MZ) and dizygotic (DZ) twins on several cognitive tasks and found that although speed of processing on individual tasks was not significantly heritable, overall speed of response across all the tasks was heritable at .456. Vernon obtained 11 response latency measures from eight tasks from 50 MZ and 52 DZ twin pairs and found that heritabilities for individual tasks ranged from .24 to .90. Within each group, correlations were submitted to a principal-factors analysis, which yielded only one factor with an eigenvalue greater than 1, accounting for 83% and 69% of the variance for the MZ and DZ groups, respectively. All variables loaded highly (.67 to .96) on these General Speed factors, which yielded a heritability of .49. Thus, in both the McGue et al. study and the Vernon study, roughly half of the variability in response latency can be attributed to genetic factors.

The evidence included in the brief review above supports the notion of a General Speed factor that (a) encompasses a range of information-processing tasks and conditions, (b) is related to individual differences in the outcome of information processing (i.e., is related to measures of intellectual ability), and (c) may be at least partially biologically determined (potentially through the efficiency of connections in neural networks). We now move to an examination of studies in the cognitive literature, using analyses based on scatter plots of response latencies from an individual as a function of response latencies of a reference group for the same experimental conditions (i.e., Brinley plots).

Individual Differences in Cognitive Speed: Individual Brinley Plot Evidence

In addition to the support for a relatively stable characteristic of cognitive speed provided by the factor analytic studies discussed above, the analysis of Brinley plots adds further support to the notion of cognitive speed (e.g., Balota & Ferraro, 1992; Charness & Campbell, 1988; Hale & Jansen, 1994; Maylor & Rabbitt, 1994; Salthouse, 1993). Hale and Jansen tested undergraduates on a battery of cognitive tasks. They found that condition means for each individual were a linear function of the condition means for all individuals. Similar results, originally reported in a slightly different form by Balota and Ferraro, are presented in Figures 3A and 3B.

Here, the fastest and slowest individuals' mean response latencies are plotted against the condition means for an average group (younger and older adults are plotted separately) drawn from the middle 50% of the distributions of overall speed. As can be seen, the performance of the fastest and slowest individuals in each group is described nicely by a linear function of the average group's condition means. The slope of the best fitting linear

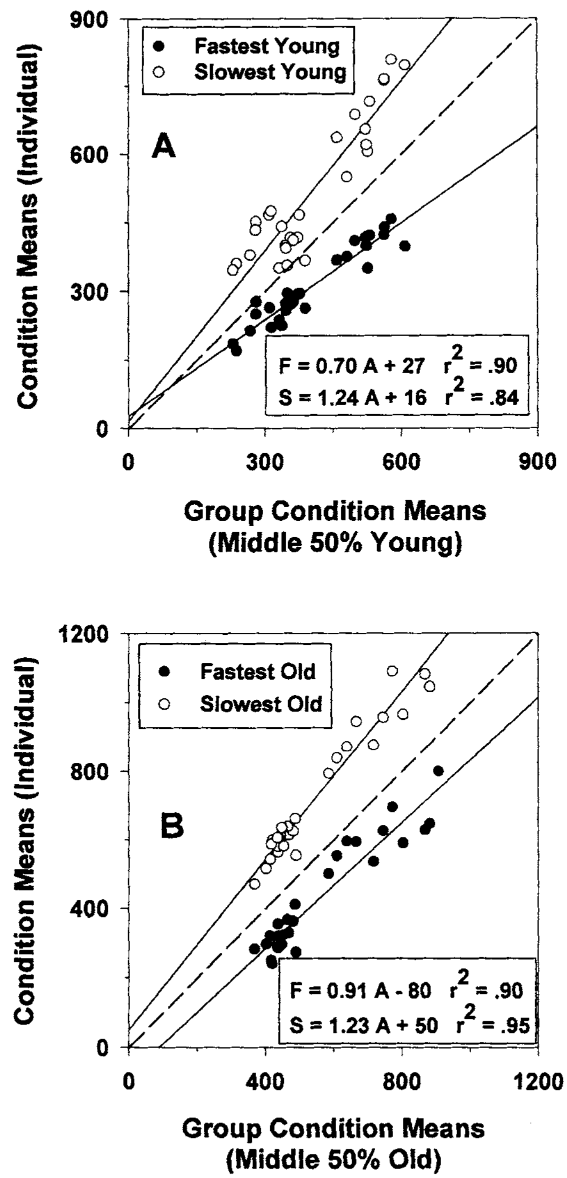


Figure 3. Scatter plots of condition means for the fastest and slowest individuals in a group as a function of the group means for the middle 50% of the rest of the individuals in that group. Panel A: Younger adults. Panel B: Older adults. F = fastest; A = average; S = slowest. Data are from "What Is Unique About Age in Age-Related General Slowing?" by D. A. Balota and F. R. Ferraro, 1992. Reprinted with permission.

function for the fastest individuals is less than unity, indicating proportionately faster speed of information processing than in the average individual. Similarly, the slope for the slowest individuals is greater than unity, indicating proportionately slower information processing than in the average individual. The results of Balota and Ferraro (1992; depicted in Figures 3A and 3B) were similar to those of Hale and Jansen (1994) and used a completely different set of cognitive tasks. Furthermore, support for a General Cognitive Speed factor was found for both younger and older adults by Balota and Ferraro.

The results depicted in Figures 3A and 3B (Balota & Ferraro, 1992), as well as those reported by Hale and Jansen (1994), are important because they provide stronger constraints with regard to individual differences in information-processing rate and amount than do factor analytic studies. For the slope of the Brinley function to remain constant across the range of response latencies, the individual must process information at a relatively similar average rate across tasks (Hale & Jansen, 1994). Factor analytic methods yielding results consistent with a General Speed factor are based on covariances, which guarantees only that an individual's standing relative to other individuals remains constant across conditions. Thus, the analysis of individual Brinley plots (e.g., Hale & Jansen, 1994) adds support to the notion of a characteristic cognitive speed that remains relatively stable within an individual across a range of tasks.

Processing Amount

We now turn to a consideration of the general information-processing requirements that are shared by most tasks typically used to probe elementary cognitive operations. These tasks are typically performed quickly and accurately by young adults. Thus, we can assume that, with few exceptions, responses are made as a consequence of the correct response becoming available to the participant (rather than guessing). Information-processing models that predict distributions of individual response latencies assume a wide range of details of information processing (e.g., Luce, 1986; McClelland, 1979; Ratcliff, 1988). Many of these models assume two very general principles: (a) that information regarding the appropriate response accumulates over time and (b) that once some sufficiency criteria are met, a response is generated.

The rate–amount model of response latencies is not based directly on the accumulation of information (sometimes called *activation*) for a particular response; rather, it is based on the idea that a certain amount of information processing must be performed to produce the information necessary to reliably choose and initiate a response. Here, we can use the terms *difficulty* and *amount of information processing* interchangeably because we assume that nearly all items would have been responded to correctly had there been no emphasis on having individuals perform the task in question quickly. Thus, items or tasks that require more processing on the part of the average individual will be referred to as more difficult, even though error rates might be similar.

The Multiplicative Rate–Amount Model of Average Response Latency

Consider a general view of events occurring within an individual following stimulus presentation. Following presentation of the stimulus, information regarding possible response outputs accrues over time as a result of cognitive processing until response criteria are met. We assume that each individual possesses a characteristic information-processing rate that is relatively constant over the time period in which measurements are made but may vary somewhat over longer time periods because of fluctuations in alertness, arousal, or motivation. Further, we assume that each item requires a certain amount of information processing before the average participant can reliably select and initiate a response. Of course, individuals differ in terms of the amount of cognitive processing performed to solve a problem, with some individuals performing more cognitive processing, on average, than the average person and others performing less cognitive processing, on average, than the average person to reliably perform a given task. Finally, once enough cognitive processing is performed to meet the response criteria, a response is then executed.

The Structural Model

Equation 2 incorporates the assumptions discussed above into a psychometric model:

$$E[L_{ij}] = \frac{\mu + \delta_j + \omega_i}{\tau_i} = \frac{\delta_j}{\tau_i} + \kappa_i. \tag{2}$$

The subscript i refers to the ith individual, and the subscript j refers to the jth experimental condition. The *expected value* of the response latency (L_{ij}) for the ith individual in the jth condition equals the ratio of the sum of the amount of information processing required (i.e., $\mu + \delta_j + \omega_i$) divided by the cognitive speed of the ith individual (τ_i). Here the amount of information processing performed by a particular individual in a particular condition is expressed as the sum of the overall average amount of processing performed by the average individual across all experimental conditions (μ), the deviation from the average processing amount due to condition (δ_j), and the deviation from the average processing amount due to individual (ω_i). Thus, the rate–amount model predicts response latencies on the basis of each individual's characteristic cognitive speed (τ_i), each individual's tendency to process either more or less information relative to others being tested (ω_i), and the difficulty of each condition expressed as a deviation (δ_j) from the overall information-processing amount (μ).

The rate–amount model can be expressed as the sum of two additive time constants (see right side of Equation 2). The first (i.e., δ_j/τ_i) represents the multiplicative change in response latency as individual is held constant and task/condition difficulty is varied. The second time constant presented in the right side of Equation 2 (i.e., $\kappa_i = [\omega_i + \mu]/\tau_i$) reflects variability in response latency that is due to individuals only, being based on the sum of a constant amount of information processing common to all individuals and tasks/conditions (i.e., μ) and the differential amount of information processing performed by an individual divided by the rate of information processing of that individual.

It is common to assume that response latencies are composed of a time component that varies with task demands and a second time component that does not vary with task demands (e.g., Cerella, 1985; Luce, 1986, chap. 3). However, the second time component is usually conceptualized as an input–output time component that remains relatively constant within an individual or group of individuals across a range of task demands (e.g., Nosofsky & Palmeri, 1997). The rate–amount model (see Equation 2) defines processing amount and cognitive speed in terms of the whole time interval between stimulus presentation and response. The time constant (κ_i) is therefore based on all information processing, both central and peripheral, occurring between stimulus and response that is common to all experimental conditions for a particular individual. One way to obtain separate estimates of central and peripheral processing parameters would be to explicitly model two additive stages.

Modeling information processing in serially ordered steps adds a level of complexity we have expressly avoided. For example, if we were to modify the rate–amount model of Equation 2 such that there were two different stages of processing, each with a different rate and amount of processing for an individual, then a simple change in the amount of information processed during one of the stages would not only decrease the overall amount of cognitive processing performed, but it would also change the overall average rate. Recently, Fisher and Glaser (1996) provided a framework for analyzing group differences in response latency under conditions in which a well-defined serial model for a particular experimental task can be specified. Although this approach certainly has its merits, one major drawback is that it relies on relatively strong theoretical assumptions that we feel many researchers will find premature, given our relatively weak knowledge of many of the tasks used in cognitive research. We therefore have chosen to use rate and amount parameters that are based on a few broad assumptions regarding how people process information and are not easily broken down into serial stages but instead are designed to describe the overall rate and amount of processing from stimulus presentation through response. Thus, our rate-amount model, as outlined in Equation 2, is best interpreted in terms of cognitive speed (τ_i) and processing amount ($\mu + \delta_j + \omega_i$) and not in terms of central versus peripheral processing.

Predictions of the Rate–Amount Model

Before fitting the rate–amount model to actual data, we first discuss the structure that the model imposes on the expected value of response latencies as a function of individuals and of experimental conditions. These predictions are presented in detail in Appendix A and are derived directly from Equation 2 using the definition of the mean and standard deviation; simple algebra; and, in some cases, linear regression.

The rate–amount model can be seen to impose a general linear structure on the expected values of response latencies in the sense that five general linear relationships are predicted in the overall structure of the response latencies. The coefficients for this general linear structure are admittedly complex (see Appendix A), but they are linear nonetheless.

Consider the case in which average response latencies for a number of experimental conditions have been gathered from a number of individuals in which all individuals participate in all conditions (i.e., repeated measures). To facilitate discussion of the five linear predictions of the rate–amount model, we depict each prediction graphically in Figures 4–7 in terms of its fit to the data of Hale, Myerson, Faust, and Fristoe (1995). More details regarding the Hale et al. study are provided in the next section.

Prediction 1: The individual Brinley function. The rate–amount model predicts that the Brinley plot of the condition means for a particular individual as a function of the overall group means for the same conditions will be linear (see Equation A2, Figure 4).

Prediction 2: Group Brinley functions. The rate–amount model predicts that the Brinley plot of the condition means for a particular group as a function of the condition means for another group will be linear (see Equation A3, Figure 5).

Both the rate–amount model and the multilayer slowing model proposed by Cerella (1985, 1990) predict a generally linear Brinley function. However, Cerella's multilayer slowing model is based on the assumption that the additive component reflects the amount of peripheral input–output information processing performed by an individual (see discussion of Equation 2 above), which is different from the assumptions underlying the additive component of the rate–amount model (κ_i in Equation 2). Thus, although the two models predict a Brinley function with a similar form, the interpretation of the intercept term in the Brinley function differs across the rate–amount and multilayer slowing models.

Prediction 3: Latencies from a condition. For a given experimental condition (i.e., holding the subscript j constant), the mean response latency for each individual is predicted to be linearly related to each individual's overall mean (see Equation A5, Figure 6).

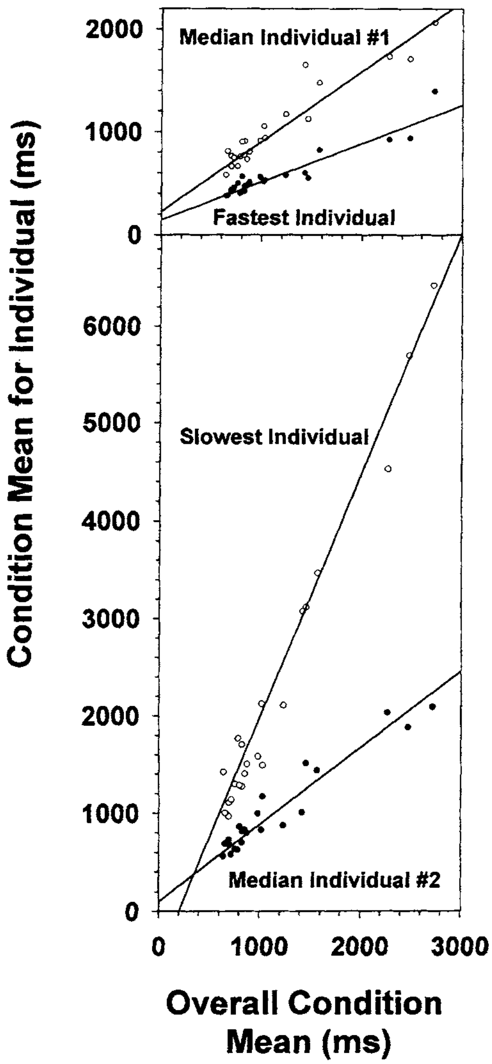
Prediction 4: Individual standard deviations and overall means. For each individual, the standard deviation across experimental conditions is predicted to be linearly related to that individual's overall mean (see Equation A7, Figure 7).

Prediction 5: Condition standard deviations and group means. For each experimental condition, the standard deviation across individuals is predicted to be linearly related to that condition's overall mean (see Equation A9, Figure 7).

Studies within the chronometric tradition have found a consistent relationship between the variability of condition means within an experimental condition (i.e., between-subjects diversity) and the overall group mean for that condition (e.g., Chapman, Chapman, Curran, & Miller, 1994; Maris, 1993a). Hale, Myerson, Smith, and Poon (1988) undertook a meta-analysis and also found that group means and between-subjects diversity for experimental conditions were linearly related (r^2 = .87 and .86 for vocal and manual responses, respectively). Young and old adults' diversity fell on the same regression lines, and more importantly, a partial correlation analysis indicated that changes in between-subjects diversity was attributable to speed, independent of age.

The Fit of the Rate–Amount Model to Data From Hale et al.'s (1995) Study

To evaluate the fit of the rate–amount model to observed response latencies, we obtained the mean latencies of each of 19 younger adults and 19 older adults for each of 22 experimental conditions (seven distinct nonverbal tasks: line-length discrimination, same–different choice, letter classification, shape classification, mental rotation, visual search, and abstract matching) previously published by Hale et al. (1995). These data provide a good test of the model because of the relatively wide range of overall average response latencies produced by the participants and the relatively broad range of task difficulties (condition means ranged from 475 ms to 1,657 ms for the younger adults). We then fitted the rate–amount model (see Equation 2) to the data from Hale et al.'s study using gamma linear model analysis (GALIMA; Maris, 1993a, 1993b).



Model Prediction

Ind.	Age	Slope	Int
Fastest	Y	.37	143
Mdn #1	Y	.69	221
Slowest	O	2.47	-491
Mdn #2	O	.78	113

Figure 4. Scatter plots of condition means for four individuals as a function of the overall mean across all individuals for the same conditions (see Prediction 1 in text). Ind. = individual; Int = intercept; Y = young; O = old. Data are from "Converging Evidence for Domain-Specific Slowing From Multiple Nonlexical Tasks and Multiple Analytic Methods," by S. Hale, J. Myerson, M. E. Faust, and N. Fristoe, 1995, *Journal of Gerontology: Psychological Sciences, 50B,* pp. 202–211. Copyright 1995 by the Gerontological Society of America. Reprinted with permission.

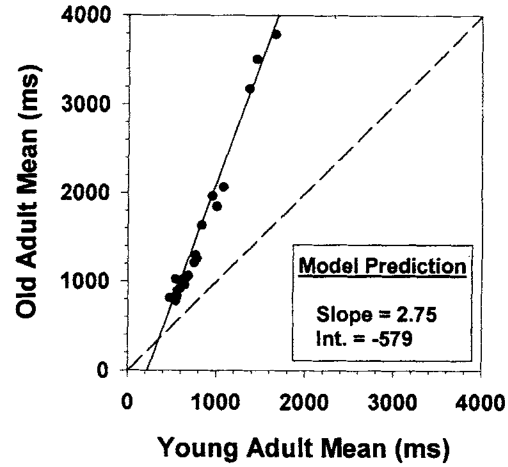
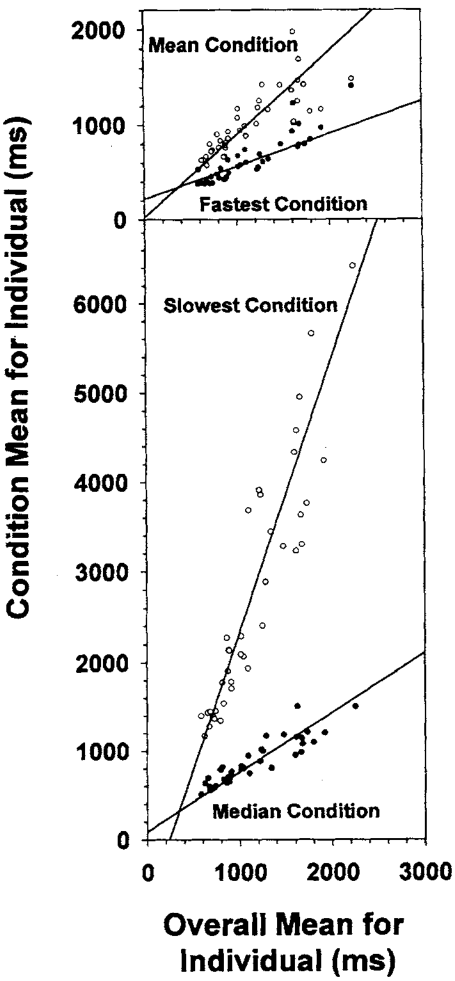


Figure 5. Scatter plot of older adult mean latency as a function of younger adult mean latency from the same experimental conditions (see Prediction 2 in text). Int. = intercept. Data are from "Converging Evidence for Domain-Specific Slowing From Multiple Nonlexical Tasks and Multiple Analytic Methods," by S. Hale, J. Myerson, M. E. Faust, and N. Fristoe, 1995, *Journal of Gerontology: Psychological Sciences, 50B,* pp. 202–211. Copyright 1995 by the Gerontological Society of America. Reprinted with permission.

GALIMA allows for fitting models in which the structure of the expected values of the dependent variable are fundamentally multiplicative in nature, as is the rate–amount model of Equation 2. The GALIMA procedure has the advantage of using maximum likelihood estimation techniques to jointly estimate parameters for each individual and experimental condition simultaneously. Because the structure of the parameters in the rate–amount model (see Equation 2 and Appendix A) is one of a large set of possible multiplicative models the GALIMA procedure can fit, and because it is the only procedure we are aware of that is designed to provide simultaneous joint estimates of all of the parameters in such multiplicative models, the GALIMA procedure provides a good starting point for evaluating the overall fit of the rate–amount model to the data. However, GALIMA is based on the assumption that the dependent variable is distributed roughly as a generalized gamma distribution, which seems to be a good rough approximation for response latency for individual responses (e.g., Luce, 1986; Maris, 1993a). This assumption is somewhat violated by the fact that the central limit theorem dictates that mean latencies are better approximated by a normal distribution. On the other hand, GALIMA seems to be fairly robust to deviations from the assumption of a generalized gamma distribution (Maris, 1993a) and under some conditions the generalized gamma distribution can appear somewhat bell shaped.

We therefore chose to include overall measures of goodness of fit from the GALIMA fitting procedure and specific linear predictions based on the GALIMA parameter estimates (see Figures 4–7). We include model predictions only to demonstrate the closeness of the linear equations developed in Appendix A to actual data for an actual set of parameter estimates. We do not mean to suggest that the rate–amount model is necessarily synonymous with GALIMA. In fact, we anticipate that under typical conditions, researchers wishing to evaluate the consistency of the rate–amount model with observed



Model Prediction

Cnd.	Task	Slope	Int
Fastest	LEN	.32	237
Mean	SRC	.87	46
Slowest	MAT	3.09	-724
Median	LTR	.66	118

Figure 6. Scatter plots of condition means for all individuals in each of four conditions as a function of the overall means for the same individuals across all conditions (see Prediction 3 in text). Cnd. = condition; Int = intercept; LEN = line-length discrimination; SRC = visual research; MAT = abstract matching; LTR = letter classification. Data are from "Converging Evidence for Domain-Specific Slowing From Multiple Nonlexical Tasks and Multiple Analytic Methods," by S. Hale, J. Myerson, M. E. Faust, and N. Fristoe, 1995, *Journal of Gerontology: Psychological Sciences, 50B,* pp. 202–211. Copyright 1995 by the Gerontological Society of America. Reprinted with permission.

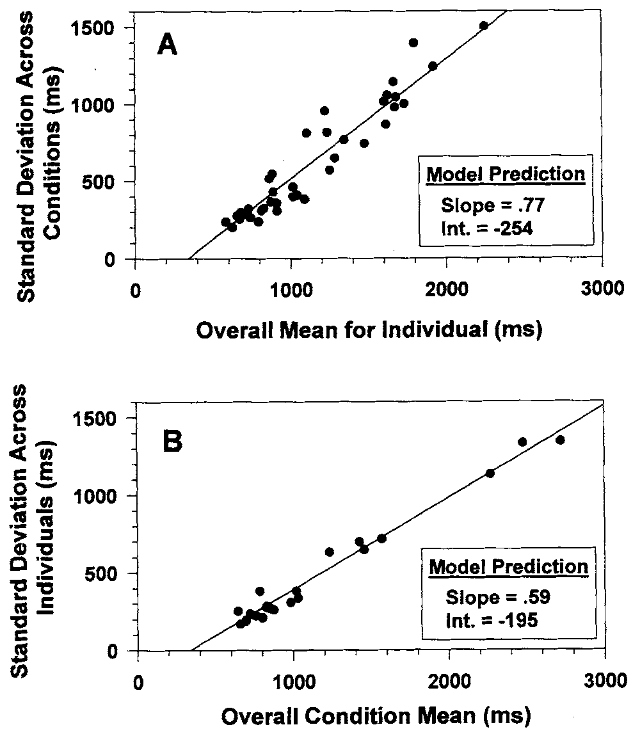


Figure 7. Scatter plots of standard deviations of mean response latencies as a function of overall means. Panel A: Standard deviation across conditions for an individual and overall mean for that individual. Panel B: Standard deviation across individuals for a particular experimental condition and overall mean (across individuals) for that same condition (see Predictions 4 and 5 in text). Int. = intercept. Data are from "Converging Evidence for Domain-Specific Slowing From Multiple Nonlexical Tasks and Multiple Analytic Methods," by S. Hale, J. Myerson, M. E. Faust, and N. Fristoe, 1995, *Journal of Gerontology: Psychological Sciences, 50B,* pp. 202–211. Copyright 1995 by the Gerontological Society of America. Reprinted with permission.

data will examine scatter plots similar to those presented in Figures 4–7 to determine if these scatter plots are well fit by linear functions.

The GALIMA yielded an R^2 statistic (i.e., a pseudo-R^2 statistic that assessed the closeness of the model to the observations relative to the closeness of a single parameter model to the observations) of .933. Even though the R^2 statistic indicates an overall high level of fit with the data, it is still informative to examine graphically (see Figures 4–7) how well the five structural predictions of the rate–amount model fit these data. Figures 4–7 clearly demonstrate that the specific linear relationships predicted by the rate–amount model provide acceptable qualitative fits for the data from Hale et al.'s (1995) study. As indicated by the R^2 statistic, nearly all of the variability in response latencies across the relatively wide range of tasks used by Hale et al. can be explained by three factors: (a) the difficulty of the experimental condition, (b) the cognitive speed of the individual, and (c) the processing-amount parameter of the individual.

Global Processing Parameters Versus Cognitive Components

Although most research in the chronometric tradition (e.g., Posner, 1978; Sternberg, 1969) has focused on identifying and isolating component cognitive operations, an increasing number of researchers have begun to examine group differences in global aspects of processing, such as overall speed (e.g., Cerella & Hale, 1994; Ferraro, 1996; Hale et al., 1993; Jensen, 1988; Kail & Salthouse, 1994; Maylor et al., 1992; Mayr & Kliegl, 1993; Myerson & Hale, 1993; Myerson et al., 1990; Nebes, Brady, & Reynolds, 1992; Nebes & Madden, 1988; Pate & Margolin, 1994; Salthouse, 1992). By providing global processing parameters for individual differences in information-processing rate and amount, the rate–amount model (as fit by GALIMA; Maris, 1993a, 1993b) holds the promise for new research into group differences in global aspects of processing. For example, in the domain of cognitive aging, does the cognitive speed of faster younger adults change more or less with age than the cognitive speed for slower younger adults? Are there systematic age-related differences in the amount of cognitive processing performed to solve a range of tasks?

The overarching goal of our analysis of response latencies is to separate global information-processing factors—which influence the large-scale structure of response latencies (on the order of 100s of milliseconds) across a wide range of individuals and conditions—from task-specific factors, which influence the small-scale structure of response latencies (on the order of 10s of milliseconds; e.g., semantic priming). An interesting question arises with regard to the factors that might cause the group differences in the global processing parameters of cognitive speed and processing amount to vary.

Evidence is accumulating indicating that group differences in cognitive speed may vary between broad domains. For example, the relationship between older adult latencies and younger adult latencies has been shown to differ within broad classes of processing, such as lexical versus nonlexical tasks (e.g., Hale et al., 1991; Lima et al., 1991), or with differential involvement of working memory (e.g., Mayr & Kliegl, 1993). These results suggest that group differences in global processing parameters should be of increasing interest to those researchers attempting to understand broad patterns of group differences in information processing. This can be accomplished by constructing Brinley functions for each task or experiment and testing them statistically.

Various strategies for detecting task-specific cognitive speed factors have been proposed (e.g., Cerella, 1991; Fisk et al., 1992; Myerson, Wagstaff, & Hale, 1994; Salthouse, 1992); however, these methods all lack provisions for repeated measures, which is the predominant design used in the chronometric literature. In addition to being inappropriate for repeated measures data, many of these methods for detecting task-specific cognitive speed factors suffer from reduced power because the number of experimental conditions, rather than the number of participants, acts as the degrees of freedom for the inference test in question. Because the number of participants is typically much larger than the number of experimental conditions in chronometric studies, an increase in power should generally result from use of a test that has degrees of freedom based on the number of participants.

To take repeated measures into account in testing the data from Hale et al.'s (1995) study for task-specific differences in cognitive speed, we used a test proposed by Faust, Balota, and Ferraro (1994; see also Hale et al., 1995), based on an approach suggested by Lorch and Myers (1990) and by Balota and Chumbley (1984), which uses the participant as its base unit of analysis. For each older adult in the Hale et al. study, we regressed mean response latencies for each task on the younger adult condition means for that same task (i.e., a task-specific individual Brinley function). From this analysis, we obtained estimates of the relative cognitive speed (i.e., the slope of the various functions) for each older adult for each task. The variability in older adults' slopes and intercepts were then used as the basis for repeated measures analysis of variance (ANOVA) tests for task-specific differences in cognitive speed and processing amount across the tasks. There were no significant task differences in either slopes, $F(4, 72) = 1.32$, $p = .270$, or intercepts, $F(4, 72) = 1.24$, $p = .303$, for the five task-specific Brinley functions tested (two of the tasks had only two conditions each and were not tested). This result is further consistent with the hypothesis that a single cognitive speed factor can explain the large-scale structure of the results from the Hale et al. study.

Although analysis of response latencies within the framework suggested by the rate–amount model will allow researchers to ask questions regarding global processing parameters, it has traditionally been the case that psychologists taking an information-processing perspective are more typically interested in small-scale effects in response latency indicative of fundamental cognitive operations. For this reason, a major goal of the present analysis was to present methods for controlling the variability in response latencies attributable to global processing parameters. This would allow for a person-by-person rescaling of response latencies so that group differences in small-scale effect sizes (i.e., presumably reflecting group differences in fundamental cognitive operations) would not be confounded by group differences in overall response latency. As discussed later, the rate–amount model may act as a guide for a linear rescaling of each individual's response latencies to a common scale that will allow for more reliable identification of group differences in component cognitive operations.

Transforming Response Latencies to a Common Scale

Two basic approaches for transforming response latencies using the rate–amount model (see Equation 2) are presented below. First, we might use some sort of estimation technique, such as GALIMA (see preceding discussion of fit of rate–amount model to data) or linear regression, to generate parameter estimates that can be then used as the basis for a transformation. Alternatively, we might use sample statistics, such as the mean and standard deviation, which might then be combined with raw response latencies (e.g., dividing by each individual's mean to produce a proportion) to form transformed observations. Both approaches can be equally useful, and both approaches share the common requirement that observed data must be rescaled linearly across individuals.

Recall that the rate–amount model predicts that each individual's response latencies will be a linear function of, and thus a linear transformation away from, the average group response. The overarching goal, then, is to perform a linear transformation of each individual's latencies to reach some common scale. Assuming one has access to unbiased estimates of the parameters of the rate–amount model (see Equation 2), rescaling can be accomplished by some combination of multiplication and subtraction using the cognitive speed and processing-amount parameters.

GALIMA Parameter Transformation

Because GALIMA allows for joint estimation of person and condition parameters from a given set of data, it should be possible to use GALIMA parameter estimates of the fit of the rate–amount model to transform latencies. Assuming that parameter estimates (e.g., t_i, $\bar{t}.$, and k_i) are unbiased independent estimates of rate–amount model parameters (e.g., τ_i [cognitive speed] and κ_i, where $\kappa_i = \omega_i/\tau_i$ [individual time constant, independent of condition]), it is possible to determine the expected value of the GALIMA-based transformation (where $\tau_i{}^* = \tau_i{}^{-1}$):

$$E[G_{ij}] = E\left[(L_{ij} - k_i)\frac{t_i}{\bar{t}.} + \bar{k}.\right] = \delta_j\overline{\tau.^*} + \bar{\kappa}., \tag{3}$$

where L_{ij} is the observed mean latency for the ith individual in the jth condition and the parameters t_i and k_i are GALIMA estimates of the rate–amount (see Equation 2) parameters for the ith individual. The result is a transformation that is linear for each individual (i.e., holding i constant) and yields an expected value that includes condition difficulty (δ_j) divided by an average (constant) speed parameter plus an additive constant. The GALIMA transformation is therefore expected to produce latencies scaled to conform to those of a participant with average rate and amount parameters (i.e., $\bar{\kappa}$ and $\overline{\tau^*}$).

Regression Transformation

Because GALIMA is not currently available in any major statistical package, and because the analysis program currently available (Maris, 1993a, 1993b) to perform GALIMA is somewhat limited in the number of individuals and conditions that can be analyzed at once, it seems prudent to consider a transformation based on parameter estimates that can be easily obtained using standard regression techniques, if possible. As demonstrated in Equation A2, the rate–amount model predicts that condition means for an individual will be linearly related to the overall means for each condition. We can therefore use simple linear regression of the overall means on each individual's latencies to derive slope and intercept parameters that indicate the individual's information-processing rate and amount relative to the "average" individual. Equation 4 presents the expected value of the transformation based on parameter estimates from simple regression:

$$E[R_{ij}] = E[L_{ij}\,b_{i1} + b_{i0}] = \delta_j\overline{\tau.^*} + \bar{\kappa}., \tag{4}$$

where $E[R_{ij}]$ is the expected value of the transformation and b_1 and b_0 are the simple regression parameters generated by regressing the overall condition means on the condition means for each individual. Of course, Equation 4 assumes that unbiased and consistent estimates of the slope and intercept parameters in Equation A2 can be obtained by simple regression. The regression transformation of Equation 4 has one advantage over the GALIMA transformation of Equation 3: The resultant overall means for each individual will all equal each other and simultaneously all be equal to the overall mean across individuals and conditions in the original untransformed latencies. The GALIMA transformation does not guarantee strict equality in this regard because of the maximum likelihood estimation technique used.

It is worth noting that Madden (e.g., Madden et al., 1992, 1993) has suggested a similar transformation that does not take individual differences into account and is applied to only one of two groups being compared. The regression transformation of Equation 4 can be seen as a generalization and extension of Madden's approach, with added theoretical motivation provided by the rate–amount model.

The regression transformation has the advantages of being simple (i.e., it simply requires multiplying the slope and adding the intercept parameters from the linear regression of an individual on everyone else) and generally available because of the widespread use of linear regression in behavioral science. The major potential weakness of both the regression and the GALIMA transformations is that they rely on estimation techniques, which in turn rely on multiple constraints in the data to yield parameter estimates. The parameter estimates for both the regression and GALIMA transformation will, of course, be sensitive to restriction of range in conditions. Moreover, the GALIMA transformation will also be sensitive to restriction of range in terms of the cognitive speed of individuals because it simultaneously estimates person and condition parameters. Therefore, although use of the regression transformation has much to recommend it, we do suggest that researchers wishing to use this technique consider designs in which participants are tested across a range of experimental conditions to ensure enough data for reliable estimation of regression parameters.

We now turn to two transformations, the z-score and proportion transformations, that do not use estimation methods. These transformations use the sample mean and standard deviation for each individual as a basis for transformation of that individual's response latencies.

Z-Score Transformation

The z-score transformation has been recently proposed as a method for controlling for a loss in power due to individual differences in overall response latency (Bush, Hess, & Wolford, 1993). Bush et al. only recommended use of the z-score transformation to remove the influence of individual differences in overall mean response latency within a single group. Nonetheless, as discussed later, to the extent that the rate–amount model provides a good fit to observed data, the z-score transformation will provide a (somewhat biased) rescaling of the data. Equation 5 presents the expected value of the z-score transformation obtained by taking each individual's condition means, subtracting their overall mean, and dividing by the standard deviation of their condition means:

$$E[Z_{ij}] = E[(L_{ij} - \bar{L}_{i.})/S_{L_i}] = \frac{(\delta_j - \bar{\delta}.)}{S_{\delta.} + \tau_i a_i} + C_{ij}. \tag{5}$$

The additive error term (C_{ij}) appears because (a) the top and bottom of the z-score ratio are not necessarily independent, (b) we are using the reciprocal of the expected value to approximate the expected value of the reciprocal, and (c) we are using Equation B10 to approximate the sample standard deviation (see Appendix B).

Constant a_i is a biasing error term (see Equation B10), which is associated with the average intrinsic variability (i.e., variability from response to response for an individual holding condition constant) for an individual. The approximation in Equation 5 reflects the fact that if there was no variability in repeated responses to the same stimuli (i.e., no intrinsic variability in individual responses in a given testing condition; all σ_{ij}s $= 0$ for all L_{ij}s in Equation A1), then the z-score transformation would directly reflect the z-score scaling of condition difficulties. In other words, in the absence of intrinsic variability, the only reason mean response latencies would vary would be individual differences in processing rate and condition differences in difficulty. Under such conditions, the z-score transformation would be a standardized scale of amount of information processing.

Unfortunately, the right side of Equation 5 includes an error term (a_i; see Equation B10), which biases the z-score transformation as a function of the intrinsic variability of the latencies of individual responses for a person performing in a given condition. Furthermore, this bias term interacts multiplicatively with the processing rate of the individual. Thus, to the extent that the product of intrinsic variability and processing rate differs across individuals, the z-score transformation will be *differentially* biased for individuals. However, to the extent that different groups do not differ in intrinsic variability, the z-score transformation will be effective.

Figure 8 presents the results of the application of the z-score transformation to the data from the Hale et al. (1995) study. Given that the Brinley function that best fits the transformed data is very close to the identity line, it appears that the z-score transformation has effectively removed group differences in cognitive speed and processing amount while avoiding problems due to differential bias across groups. Under such conditions, the z-score transformation provides an easy-to-use and easy-to-interpret rescaling of latencies to a standardized scale of information-processing amount.

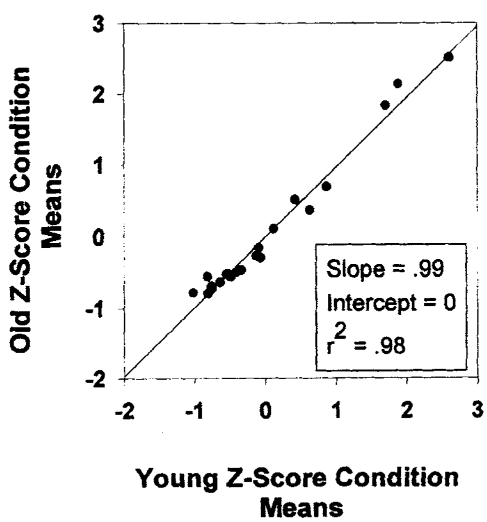


Figure 8. Scatter plot of z-score-transformed latencies for older adults as a function of z-score-transformed latencies for younger adults for the same experimental conditions. Data are from "Converging Evidence for Domain-Specific Slowing From Multiple Nonlexical Tasks and Multiple Analytic Methods," by S. Hale, J. Myerson, M. E. Faust, and N. Fristoe, 1995, *Journal of Gerontology: Psychological Sciences, 50B*, pp. 202–211. Copyright 1995 by the Gerontological Society of America. Reprinted with permission.

Although there are no immediately apparent group differences in the global structure of the transformed data depicted in Figure 8, there may still be group differences in small-scale structure. That is, a repeated measures ANOVA may still reveal that some of the deviations from the identity line in Figure 8 are due to systematic group differences. Before applying such an analysis to the data of Hale et al. (1995), we present one final transformation for comparative purposes.

Proportion Transformation

Several recent studies of cognitive aging have used a proportion transformation of response latencies in which each condition mean for an individual is divided by some measure of overall response latency (see Chapman et al., 1994; Charness & Campbell, 1988; Dulaney & Rogers, 1994; Hartley, 1993; and Spieler et al., 1996, for recent examples). This is often justified by assuming that individuals differ by being proportionately slower or faster than each other. If such is the case, then dividing each individual's condition mean for a given experimental condition by their overall mean should yield a value that remains constant across individuals. That is, constant experimental effects should yield constant proportions in response latencies across individuals.

Assuming that condition means for one individual are proportional to the condition means for another is equivalent to assuming that a Brinley plot of the condition means will be best fit by a line that passes through the origin. As already noted, this is often not the case (see Figure 4 and Table 1). Therefore, the assumptions underlying ratio transformation will often be violated in real data.

Equation 6 presents the expected value of the proportion transformation:

$$E[P_{ij}] = E\left[\frac{L_{ij}}{L_{i.}}\right] = \frac{\tau_i^*\delta_j + \kappa_i}{\tau_i^*\delta_. + \kappa_i} + C_{ij}. \qquad (6)$$

First, note that the additive error term (C_{ij}) appears because the top and bottom of the ratio in the middle term of Equation 6 (i.e., the proportion transformation) are not necessarily independent and because we used the reciprocal of the expected value to approximate the expected value of the reciprocal. Although the additive error term (C_{ij}) will not be negative, its exact value also depends on the sampling distributions of the specific variables measured (i.e., L_{ij}).

Also, note that Equation 6 demonstrates that the proportion transform will only yield an expected value that is the proportional difficulty of each condition (i.e., what one might intuitively expect the transformation to measure) when the additive constants (i.e., κ_i and C_{ij}) in Equation 6 are zero. As can be seen in the right-hand side of Equation 6, taking each individual's mean latency in each condition and dividing by that individual's overall mean latency has an expected value that is a biased measure of the proportional difficulty of a particular condition. The bias is represented by two additive constants (i.e., κ_i and C_{ij}) and is due to problems associated with taking the expected value of a ratio and to individual differences in the processing-amount parameter (κ_i). It can be shown that when the average processing-amount parameters for two groups are equal and the distributional assumptions for the two groups are equivalent, the rate–amount model yields a Brinley function that is approximately proportional (i.e., linear and goes through the origin). Under such limited conditions (see Mayr & Kliegl, 1993, for an example), the proportion transformation will provide an effective rescaling of latencies to a common scale. However, if the Brinley function does not go through the origin for real data, this is an indication that the proportion transformation is most likely inappropriate.

Figure 9 presents the results of the application of the proportion transformation to the data from the Hale et al. (1995) study. As can be seen, the linear Brinley function that best fits the transformed data deviates markedly from the identity line, indicating a poor rescaling. This result was to be expected, given that the linear Brinley function fit to the raw data (see Figure 5) does not go through, or even near, the origin. Thus, the proportion transformation appears to be inappropriate for the data of Hale et al.

Finding Task-Specific Group Differences in the Presence of Global Group Differences

Examination of Figure 5 might lead to the conclusion that it is unlikely that any task-specific group differences might be found in the data from Hale et al.'s (1995) study. After all, 98% of the variability in older adult condition means is accounted for by younger adult condition means. The problem with such a conclusion is that it neglects to take scale differences into account. The Brinley plot depicted in Figure 5 displays the large-scale structure of the data across the whole range of individuals and conditions. However, if we imagine zooming in our view to examine just two or three conditions at a time, then the deviations from the global relation will be "blown" up. What we are attempting by transforming all individuals to a common scale is a rescaling of the raw deviations from a common linear Brinley function so that we can appropriately use an ANOVA to evaluate these small-scale effects.

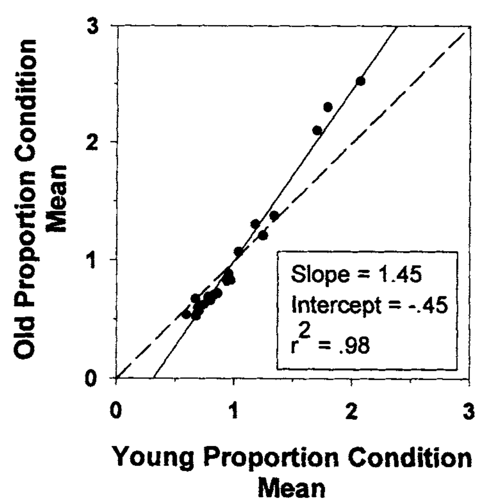


Figure 9. Scatter plot of proportion-transformed latencies for older adults as a function of proportion-transformed latencies for younger adults for the same experimental conditions. Data are from "Converging Evidence for Domain-Specific Slowing From Multiple Nonlexical Tasks and Multiple Analytic Methods," by S. Hale, J. Myerson, M. E. Faust, and N. Fristoe, 1995, *Journal of Gerontology: Psychological Sciences, 50B*, pp. 202–211. Copyright 1995 by the Gerontological Society of America. Reprinted with permission.

To demonstrate that it is possible to find small-scale group differences in response latencies even in the presence of a strong single general-slowing component, we reanalyzed the data from the Hale et al. (1995) study using raw latencies, z-score-transformed latencies, and regression-transformed latencies. We then compared the ANOVA results of all three forms of the data with regard to Group × Condition interactions in each of the seven tasks included in the Hale et al. study. Table 3 displays this comparison.

As can be seen, six of the seven tasks yielded a Group × Condition interaction in the raw latencies, but this is to be expected, given the good fit to a linear group Brinley function with a slope different from unity (see Figure 5). We therefore suspect that most of these interactions may be spurious (i.e., due to linear group differences in scaling of response latencies). Consideration of the analyses of the regression- and z-score-transformed means (see the middle and right columns of Table 3) bears this concern out.

Here, we get a consistent picture of a significant Group × Condition interaction for the letter classification task and a marginal Group × Condition interaction for mental rotation. The results depicted in Table 3 provide evidence that removing large-scale linear dependencies among individuals (by a linear transformation to a common scale) does not preclude finding small-scale Group × Condition interactions. Here, we have identified Group × Condition interactions in the letter classification task, which may indicate group differences in some fundamental cognitive operation associated with this task. Further detailed analysis, and possibly further experimentation, is needed to determine the specifics of how we should interpret this finding.

In essence, we have linearly rescaled the data from individuals so that all individual Brinley functions (i.e., the plot of condition means from an individual as a function of overall condition means) fall on the identity line and then analyzed the residuals away from these individual Brinley functions for systematic group differences caused by manipulation of the independent variables. Thus, our approach can be seen as hierarchical in that we have controlled for global linear differences in response latency and we are concerned with finding statistically significant variability on a smaller scale in the rescaled residuals.

Transforming to a Common Scale Using Individual Response Latencies

We began our discussion of individual differences in information-processing rate and amount with the example of two individuals performing a priming task with only two conditions. It is not uncommon in the chronometric literature to have tasks in which all individuals are tested in a control condition and an experimental condition, with the resultant difference being thought of as an amount (e.g., semantic priming; Myerson et al., 1992). We have argued thus far that for this to be the case all individuals must be measured on the same information-processing scale. We have suggested that estimation techniques may be used to estimate the rate–amount model parameters of cognitive speed and processing amount and that when there is not enough data to obtain reliable parameter estimates, the z-score transformation may be used. The z-score transformation appears to be widely applicable because it predicts a generally linear Brinley function. However, the z-score

Table 3
F Value for Interaction, Degrees of Freedom, and p Value for Mean RTs From Hale et al.'s (1995) Study and for Regression- and Z-Score-Transformed Means

Task	Mean RT	Regression score	z score
Line-length discrimination			
F	10.36	0.20	0.23
dfs	1, 36	1, 36	1, 36
p	.003	.660	.635
Disjunctive choice RT			
F	1.94	0.02	0.03
dfs	1, 36	1, 36	1, 36
p	.173	.896	.858
Letter classification			
F	7.28	4.54	4.41
dfs	2, 72	2, 72	2, 72
p	.001	.014	.016
Shape classification			
F	3.18	1.09	1.08
dfs	2, 72	2, 72	2, 72
p	.047	.343	.344
Mental rotation			
F	8.36	2.15	2.08
dfs	3, 108	3, 108	3, 108
p	<.001	.098	.107
Visual search			
F	19.96	0.49	0.43
dfs	3, 108	3, 108	3, 108
p	<.001	.693	.734
Abstract matching			
F	20.47	0.94	1.04
dfs	3, 108	3, 108	3, 108
p	<.001	.426	.376

Note. RT = reaction time.

transformation, as presented above (see Equation 5), involves division by the standard deviation of condition means for each individual. A problem arises in the extreme case of only two experimental conditions in which the standard deviation of each individual's condition means and the difference between those means are perfectly correlated. Thus, just as one cannot apply linear regression to two points, one cannot apply the z-score transformation of condition means to the case of only two experimental conditions.

To solve this problem, we may simply extend the z-score transformation to individual response latencies rather than to condition means as before. The logic of this extension is straightforward. Whereas before we had a difficulty parameter (i.e., δ_j) for each experimental condition, we now have a difficulty parameter (δ_j) for each item, with the index j now indexing the jth item. One potentially serious issue does appear, however, when one considers the notion that the difficulty (i.e., amount of cognitive processing required) of an item will vary with the sequential context (e.g., what response was made to the preceding item, what was the identity of the preceding item, or when was the preceding item presented?). It has long been known that sequential effects greatly influence response latencies (e.g., Morton, 1979; Rabbitt, 1966; Soetens, Boer, & Hueting, 1985).

It is rare that individual items are presented to individual participants in exactly the same order. Furthermore, whereas error rates are typically low in the class of tasks we have attempted to model, the pattern of errors across items will differ across participants. Thus, sequential effects are typically confounded with individuals. However, given that it is common to have a relatively large number of trials (e.g., in the 200 to 300 range) and that trial order is often randomized for each individual, sequential effects are likely to average out in the long run. In the analysis that follows, we assume that sequential effects average out for the most part and that when they do not, they merely add random error.

Monte Carlo Simulations

We ran several Monte Carlo simulations to evaluate the performance of analyses of raw and transformed latencies under conditions of relatively small amounts of data. We were interested both in relative power to detect within-subject effects and protection against Type I errors with regard to Group × Condition interactions. We set up a hypothetical experiment in which two groups of hypothetical individuals (20 per group) were tested in each of two experimental conditions (20 trials per condition). We randomly selected response latencies from population distributions whose means and standard deviations varied systematically in accordance with the rate–amount model (see Equation 2), thereby modeling individuals as if their response latencies were all linear transformations of each other.

Method. For each run of each simulation, a response latency distribution was generated for each individual and condition combination. Although there is no agreement in the literature regarding the correct distribution for response latencies (Luce, 1986), good empirical fits have been consistently obtained using the ex-Gaussian distribution (e.g., Hockley, 1984; Ratcliff, 1978). Because we expected the results of the simulations to be relatively stable across minor variations in positively skewed distributions, we were not concerned with finding an optimal model of latency distributions and settled on the ex-Gaussian distribution because of its popularity in the literature and ease of implementation. Therefore, each distribution was modeled as being ex-Gaussian with a modest (1.5) level of positive skew.

The means and standard deviations of these distributions varied in accordance with the rate–amount model. The parameters setting this large-scale structure (i.e., condition difficulty and the rate and amount of processing on the part of individuals) were drawn randomly for each simulated experiment within constraints suggested by the actual parameters obtained in our earlier GALIMA fit of the rate–amount model to data from Hale et al.'s (1995) study. Thus, in effect, we simulated younger and older adults' performance on a two-condition task of average difficulty based on the data from the Hale et al. study.

For each run, 20 responses were drawn at random from each latency distribution, proportion and z-score transformations were computed for each simulated latency, and condition means were computed for each individual using the raw and transformed latencies. These means were then subjected to a dependent-group t test to test for mean differences (i.e., within subject) in the experimental conditions; an independent-group t test on difference scores to determine Group × Condition interactions; and in the case of the raw means, an independent-group t test of group differences in overall means. All t tests were performed with an alpha of .05.

Eight simulations were run, with 100,000 replications each. We varied four aspects across simulations. First, we varied how much error variability was included in the latency distributions. The rate–amount model hypothesizes two sources of within-distribution variability of response latencies: (a) variability due to the difficulty of the items presented and (b) error variability. Within each simulation, we assumed a constant variance for item difficulties for both conditions and a constant error variability across all individuals and conditions. However, in half the simulations (low error) the error variability was one fourth as large as the item variability. For the other half of the simulations (high error), we kept the same item variability but increased the error variability to be of equal magnitude. Second, we varied whether or not the groups differed in average cognitive speed parameters. Third, we varied whether or not the groups differed in average processing-amount parameters. Fourth, we varied the size of the condition effect (i.e., the difference in condition difficulty), expressing it as a function of distribution standard deviation, as is customary (e.g., Bush et al., 1993).

Results and discussion. Table 4 displays the results of four simulations in which no group differences in either average cognitive speed or processing-amount parameters were modeled. We varied the amount of error variability (high or low) and the size of the within-subject effect (i.e., either none or .10 *SD*s). The main finding was that, as Bush et al. (1993) suggested, *z*-score-transformed response latencies are likely to provide greater power in detecting effects on repeated measures factors (i.e., within-subject effects; see far-right column of Table 4). We have therefore replicated and extended Bush et al. results by adding large-scale structural constraints predicted by the rate–amount model, and present in real data (e.g., Hale et al., 1995), to a Monte Carlo simulation of power to detect within-subject effects.

Table 5 displays the results of four simulations in which we did simulate a faster and a slower group, in terms of the average cognitive speed parameter, as well as group differences in the average processing-amount parameter. For these simulations, the condition effect was set at .50 *SD*s. As shown in Table 4, under conditions of no group differences in average processing-amount parameters, the proportion and *z*-score transformation analyses provided equivalent control over Type I errors (i.e., rejection of the null hypothesis approximately 5% of the time) and provided markedly more control over Type I errors than did analyses of raw latencies. However, when group differences in average processing-amount parameters (i.e., group differences in the average additive time constant; see Equation 2) were included, the *z*-score transformation analyses performed far better than did either the raw latency or proportion transformation analyses (see middle column of right half of Table 5).

Table 4
Proportion of Null Hypothesis (α = .05) Rejections for Monte Carlo Simulation of Two Groups (n = 20 Each, 20 Observations Per Condition) With Identical Information-Processing Parameters

	ES[b] = 0			ES[b] = .10		
Error variability[a]	Group	Interaction	Within subject	Group	Interaction	Within subject
High error						
RT	.051	.050	.049	.051	.050	.782
Proportion	NA	.050	.050	NA	.051	.793
z	NA	.050	.050	NA	.051	.807
Low error						
RT	.050	.051	.048	.050	.050	.588
Proportion	NA	.051	.049	NA	.051	.597
z	NA	.052	.048	NA	.051	.614

Note. ES = effect size; RT = reaction time; NA = not applicable.
[a] The intrinsic response variability of a simulated individual. [b] Difference between means of condition distributions expressed in number of standard deviations.

Conclusions: Monte Carlo simulations. The results of these simulations clearly suggest that when there is not enough data to fit the rate–amount model directly, traditional analyses of raw latencies should, in general, be augmented by a follow-up analysis of *z*-score-transformed latencies. Such an analysis should provide greater power in detecting within-subject effects and provide greater protection against acceptance of spurious Group × Condition interactions.

It is worth noting, however, that the rate–amount model predicts that the *z*-score transformation will result in unequally biased rescaling when there are group differences in the relationship between the condition means and condition standard deviations for an individual. We therefore suggest further that the standard deviation of individual response latencies be regressed on condition means and the resulting functions be compared across groups. Group differences in these functions would indicate fundamental differences in the relationship between cognitive speed and intrinsic variability across groups.

General Discussion

Studies using response latency to probe fundamental cognitive processes often treat the measurements taken on different individuals (or groups) as if they came from a physical scale with a common unit of measurement. This is unlikely to be the case with regard to underlying information processing if individuals (or groups) differ in terms of cognitive speed and/or processing amount. Our analysis of the results of Hale et al.'s (1995; see also Balota & Ferraro, 1992; Hale & Jansen, 1994) study suggest a large-scale structure for response latencies that simultaneously provides multiple linear constraints (see Equations A2–A9).

A Hierarchical View

Of central interest is the fact that the latencies for various individuals are linearly interrelated on a global scale (see Figure 4). Such linear interrelationships lead naturally to the prediction that when groups differ in terms of overall mean response latency, overadditive interactions (i.e., the slower group producing a reliably greater effect than the faster group) are to be expected. Given the increasing number of studies in the literature reporting groups that are systematically slowed in relation to some reference group (e.g., Cerella et al., 1980; Ferraro, 1996; Hale et al., 1991, 1993; Kail, 1991, 1992; Maylor et al., 1992; Nebes & Brady, 1992), it is of some concern that spurious Group × Condition interactions may be finding their way into the literature (e.g., Salthouse, 1992).

We have taken the position that it is only those experiments that provide evidence of small-scale deviations from this large-scale norm that are truly of interest for identifying group differences in specific cognitive processes (Burke et al., 1987; Kliegl & Mayr, 1992; Madden et al., 1993; Salthouse, 1992). However, our analysis also suggests that these small-scale deviations should be rescaled on a person-by-person basis according to individual-based global processing parameters.

Consideration of linear individual differences in response latencies leads directly to the view that response latencies can be modeled as the result of a certain amount of information processing performed at a given average rate (i.e., cognitive speed). We have proposed a rate–amount model, based on general

Table 5

Proportion of Null Hypothesis (α = .05) Rejections for Monte Carlo Simulation of Two Groups (n = 20 Each, 20 Observations Per Condition) With Different Information-Processing Parameters (Slower and Faster Groups) and Constant Effect Size (.50)

	Additive constant differential[b] = 0			Additive constant differential[b] = 2		
Error variability[a]	Group	Interaction	Within subject	Group	Interaction	Within subject
High error						
RT	1.000	.999	1.000	1.000	.999	1.000
Proportion	NA	.051	1.000	NA	.688	1.000
z	NA	.052	1.000	NA	.053	1.000
Low error						
RT	1.000	.987	1.000	1.000	.987	1.000
Proportion	NA	.049	1.000	NA	.499	1.000
z	NA	.051	1.000	NA	.051	1.000

Note. RT = reaction time; NA = not applicable.
[a] The intrinsic response variability of a simulated individual. [b] Group difference in the additive time component (see Equation 2) expressed in number of standard deviations.

information-processing principles, that is consistent with the large-scale structure often found in response latencies and that can be used as a basis for development of methods to linearly transform individuals' latencies to a common scale. The rate–amount model also provides a basis for identification of global processing parameters that may help researchers to better understand global group differences in information processing.

Individual and Group Differences in Response Latency

One important contribution of the present analysis is to draw the attention of those researchers interested in group differences in response latencies to the fact that the systematic group differences in response latencies bear some resemblance to systematic differences found between individuals (see Balota & Ferraro, 1992, and Hale & Jansen, 1994, for discussion of these issues within the domain of cognitive aging). To say that one group is generally cognitively slowed in relation to another presumes that each individual has a general cognitive speed and that this cognitive speed differs not only across individuals but also across groups. In our view, it is the contrast between individual differences and group differences in global processing parameters that will provide potentially interesting questions for further research. For example, are slower younger adults slowed, in terms of general cognitive speed, by the same amount as faster younger adults as they age?

Transforming to a Common Scale

The analyses presented above use the rate–amount model as a measurement model for mean response latencies and yield some clear recommendations with regard to linearly transforming the latencies of individuals to a common scale. First, if enough data has been gathered from a wide enough range of individuals and conditions, methods such as GALIMA or linear regression should be used to obtain estimates of each individual's global processing parameters. These can then be used as the basis of a linear transformation of raw latencies. Although it is beyond the scope of this article to provide a Monte Carlo–type simulation of the general performance of such transformations, both the GALIMA (Maris, 1993a, 1993b) and linear regression techniques discussed above are based on robust estimation techniques and we feel confident that they avoid the somewhat biased results obtained when the linear transformation is based solely on the results from a single individual, as are the proportion and z-score transformations.

The second major result of our analyses with respect to potential transformations is the general superiority of the z-score transformation over proportions. The results of our Monte Carlo simulations are in accord with those of Bush et al. (1993) in indicating that z-score-transformed latencies generally provide greater power for detecting within-subject effects than do proportion-transformed latencies. Our results indicate that this is the case even when the large-scale constraints of the rate–amount model are applied to the simulation. Moreover, z-score-transformed latencies provide proper control over Type I errors for Group × Condition interactions over a wider range of conditions than do proportion-transformed latencies. Proportion-transformed latencies are inappropriate whenever linear Brinley functions fail to go through the origin. As Table 1 and Figure 5 demonstrate, it is often the case that linear Brinley functions have a sizable negative intercept. Z-score-transformed latencies, however, are appropriate as long as the Brinley function is generally linear. Of course, our analysis also suggests that z-score-transformed latencies should be used with caution when the relationship between the means and standard deviations of response latency distributions (i.e., the distribution of individual response latencies for a particular participant in a particular condition) varies across groups. Because of the bias in the z-score transformation (see Equation 5), this could result in the z-score-transformed latencies being unequally biased across groups.

Regardless of which transformation method is applied, we caution researchers not to forget that the results of inference tests applied to transformed data are interpretable only in terms of the transformed dependent variable (in this case, response latency or time). Thus, analyses of transformed latencies should always be performed in conjunction with similar analyses performed on raw latencies. In fact, interesting information can be gained from any discrepancies between analysis of raw and transformed latencies.

A further caution is warranted with respect to interpretation of transformed measures. The scope of the data at hand will determine the generality of the linear rescaling. If one has data from a wide range of tasks with several experimental conditions (and a wide range of individuals) that are well fit by the rate–amount measurement model (i.e., inspection of scatter plots indicates the data have the predicted linear constraints), then application of the rate–amount model to estimate global processing parameters to be used as a basis for transforming to a common scale (or, alternatively, z-score transformations) will result in controlling for individual differences in a more general cognitive speed parameter than will similar transformations based on data from a more limited range of tasks, conditions, and individuals. A transformation based on data from a single task should be viewed conservatively as controlling for only task-specific cognitive speed.

Other Methods for Correcting for Group Differences in Overall Latency

Three other approaches to correcting for group differences in overall response latency should be discussed at this point. First, Madden (e.g., Madden et al., 1992, 1993) has proposed a linear regression transformation that is similar to our proposed transformation. In fact, our regression transformation can be viewed as a generalization of the approach taken by Madden et al., with the addition of the rate–amount model as a theoretical motivation. Madden's approach does differ somewhat from ours in that the group Brinley function is used rather than the inverse of the individual Brinley function (see Equation 4); therefore, individual differences variability is not taken into account. Furthermore, Madden's transformation is applied only to the control group to produce a simulated experimental group, which is then statistically compared with the raw latency results from the actual experimental group. Our approach is to transform all latencies to a common scale and compare analyses of raw and transformed latencies.

A second approach to correcting for group differences in overall response latency that has been used in the literature is the log transformation (e.g., Bush et al., 1993; Madden, 1990). The log transformation works by changing a constant proportion to an additive constant. That is, the log-transformed latencies from two individuals whose latencies differ by a constant proportion of 2 (i.e., when one individual always takes exactly twice as long to respond as the other) will result in transformed latencies that differ by an additive constant. Because traditional repeated measures ANOVA models handle such additive differences easily, the log transformation is a candidate for correcting proportional differences. However, there are several problems with the log transformation that call its appropriateness into question. First, it is a nonlinear transformation; thus, it represents a nonlinear rescaling of the dependent measure, which could seriously compromise interpretability (e.g., Townsend, 1992). Second, our analysis suggests that the proportion transformation (see Equation 6) will provide an unbiased linear rescaling if there truly are proportional differences between individuals. Third, because many empirical linear Brinley functions published to date in the literature fail to go through the origin (i.e., are not proportional; see Table 1 and Figure 5), the log transformation will often be inappropriate.

A third method for correcting for group differences in response latencies involves applying a specific information-processing model to the results of a specific task (Fisher & Glaser, 1996). This contrasts with our approach of applying a general latent trait-type model across all tasks. Fisher and Glaser proposed that a specific latent network model of the fundamental cognitive processes required to perform a particular task be chosen (Fisher & Glaser, 1996, confine themselves to analyses of stochastic Program Evaluation and Review Technique networks [Schweickert, 1978]). Response latency predictions can then be derived from the latent network model, and process-specific parameters can be generated for each group. Statistical tests may then be applied to see if the process-specific latent network model fits the observed data better than some common-process model (i.e., a model that explains group differences in terms of equivalent changes across groups in all processes associated with a task).

The analysis that Fisher and Glaser (1996) presented can be seen as similar to ours with respect to identifying process-specific group differences in information processing. Both the latent network model approach and our common-process rate–amount model approach depend on finding systematic group differences in response latencies above and beyond those predicted by a model of common-process group differences in information processing. The rate–amount rescaling approach presented in the present article differs from that of Fisher and Glaser in that it depends on traditional repeated measures ANOVA methodology to detect deviations from a common-process model, whereas the approach taken by Fisher and Glaser depends on specification of a specific network model of each task/condition. Fisher and Glaser's approach also differs from the rate–amount rescaling approach in that the common-process model they use as the starting point in their analysis predicts a linear group Brinley function that is proportional (i.e., goes through the origin), which, as we have shown (see Table 1 and Figure 5), is typically not the case. Furthermore, the statistical test proposed by Fisher and Glaser does not take repeated measures into account and may therefore suffer from an inflated Type I error rate, leading to inappropriately inflated rates of acceptance of spurious Group × Condition interactions (e.g., Lorch & Myers, 1990). Thus, although Fisher and Glaser's approach holds much promise for the future, it is unclear at this time if the approach as presented will be useful to the majority of researchers in the field because of its increased requirements in terms of experimental design and task-specific model fitting and the need for a more appropriate inferential test.

Processing Rate and Amount

Rate–amount models of information processing associated with specific tasks have a long history (Luce, 1986). Well-known examples include models of choice reaction time (Hick, 1952), mental rotation (e.g., Cooper & Shepard, 1973; Shepard & Metzler, 1971), short-term memory scanning (e.g., Sternberg, 1975), and visual search (e.g., Atkinson et al., 1969). However, these models have not typically included latent variables for rate and amount but rather have concentrated on factors such as numbers of response alternatives or memory set size, which could be explicitly manipulated in experiments. We have argued that the large-scale linear structure of response latencies in the 0- to 2,000-ms range across wide ranges of tasks, experimental conditions, and individuals is evidence for general latent variables of cognitive speed and processing amount. In an effort to provide a model that predicts a generally linear structure for response latencies (see Equations 3–8), we have not argued for the most general rate–amount model possible (see Equation 2). The most general model would include the sum of person, condition, and interaction terms for the amount of cognitive processing performed divided by the sum of person, condition, and interaction terms for cognitive speed. Although GALIMA can indeed provide an inferential statistical framework for analysis of this more general model, such a model is inherently nonlinear and far more complicated to analyze than the rate–amount model we have proposed. We do feel, however, that researchers who become comfortable with the rate–amount model as proposed in the present article will find extensions of the model within the GALIMA framework to be useful.

Conclusion

The rate–amount model provides a measurement model for mean response latencies that allows for identification of global cognitive speed and processing-amount parameters, which may provide useful information with regard to group differences in global information processing. Identification of global processing parameters also allows for the linear transformation of each individual's response latencies to a common information-processing scale. In the absence of enough data to effectively estimate global processing parameters with the proposed regression transformation, the z-score transformation can provide an approximate rescaling. Such rescaling can be seen to be equivalent to the rescaling of residuals from individual Brinley functions (see Figure 4). Analyses of rescaled data may then be analyzed within a traditional ANOVA framework and used to augment analyses of untransformed response latencies. We have shown that small-scale systematic variability in the rescaled latencies can be effectively detected even when, as is the case of Hale et al. (1995), the data are well fit by a single linear Brinley function. Comparing the results of analyses of raw and transformed latencies should help researchers to better identify and interpret Group × Condition interactions.

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(Appendixes follow)

Appendix A

Specification of the General Rate–Amount Model of Mean Response Latency

We now specify the general rate–amount model of mean response latency and present equations detailing key predictions this model has for the large-scale structure of mean latencies across individuals and experimental conditions. In developing this model, we assume that there are enough observations included in each observed mean latency for the sampling distributions of the mean latencies to be well approximated by a normal distribution (i.e., the central limit theorem holds). We also assume unequal variances for these sampling distributions, which leads to a situation in which some of the predictions derived from the model will be approximations because no known solution exists for some of the resultant distributions of the combination of random variables with unequal variances. We include the assumption of unequal variances in the anticipation that the next level of development of the rate–amount model will include theoretically motivated arguments regarding the relationship between the intrinsic variability within an individual and mean latency for that individual for a particular condition. Such relationships have been reported in the literature (e.g., Myerson, Widaman, & Hale, 1991; Myerson, Zheng, & Hale, 1994; Myerson, Zheng, Hale, Jenkins, & Widaman, 1999).

Model Specification

We assume I individuals participate in J experimental conditions in which multiple response latencies are measured for each condition and mean response latency (L_{ij}) in each condition is the dependent variable of interest. The rate–amount model represents response latencies in terms of information-processing amount performed by an individual in a particular condition ($\mu + \delta_j + \omega_i$) divided by a constant information-processing rate (τ_i, indicating individual differences in general efficiency of processing) for that particular individual. Information-processing amount is represented by three additive components: the processing amount common to all individuals and conditions (μ), the differential processing amount required for the average individual to reliably respond in a particular condition (δ_j, indicating the differential difficulty of a condition), and the differential processing amount performed by a particular individual across all tasks (ω_i, indicating individual differences in peripheral processes, such as response initiation, and also in central processes, such criterion for response).

As can be seen in the far right of Equation A1, the assumptions above result in a model in which the mean latency for the ith individual in the jth condition can be expressed as the sum of (a) the ratio of overall information-processing amount (δ_j) required to reliably make the correct response to the information-processing rate (τ_i) of the individual, (b) an additive constant attributable to each individual ($\kappa_i = [\mu + \omega_i]/\tau_i$), and (c) a random error term (E_{ij}) that is normally distributed with a mean of zero and a unique variance ($N[0, \sigma_{ij}]$).

$$L_{ij} = \frac{\mu + \delta_j + \omega_i}{\tau_i} + E_{ij} = \frac{\delta_j}{\tau_i} + \kappa_i + E_{ij},\ i = 1..I,\ j = 1..J. \quad \text{(A1)}$$

We assume further independence of the error terms such that all covariances are equal to zero.

Predictions of the Rate–Amount Model

The rate–amount model can be seen to impose a general linear structure on the expected values of response latencies in the sense that five general linear relationships are predicted in the overall structure of the response latencies. The coefficients for this general linear structure are admittedly complex, but they are linear nonetheless.

Prediction 1: The Individual Brinley Function

The rate–amount model (see Equation A1) predicts that the Brinley plot of the expected values of the condition means for a particular individual as a function of the expected values of the overall means for the same conditions will be linear. This can be seen by holding the i subscript (i.e., individual) constant and varying the j subscript (i.e., condition) in Equation A2:

$$E[L_{ij}] = \left(\frac{\tau_i^*}{\bar{\tau}_.^*}\right) E[\bar{L}_{.j}] + \left[\kappa_i - \left(\frac{\tau_i^*}{\bar{\tau}_.^*}\right)\bar{\kappa}_.\right], \quad \text{(A2)}$$

where $\tau_i^* = \tau_i^{-1}$.

Prediction 2: Group Brinley Functions

The group Brinley function predicted by the rate–amount model can be derived directly from the predicted individual Brinley function (see Equation A2). Prediction 2 is therefore a simple extension of Prediction 1. To derive the between-groups Brinley function (i.e., the function relating the condition means for one group as a function of those of a second group) predicted by the rate–amount model, we first assume two groups, A and B. We find that the rate–amount model predicts that the expected values of the condition means for Group A is a linear function of the expected values of the condition means for Group B:

$$E[\bar{L}_{Aj}] = \left(\frac{\bar{\tau}_A^*}{\bar{\tau}_B^*}\right) E[\bar{L}_{Bj}] + \left[\bar{\kappa}_A - \left(\frac{\bar{\tau}_A^*}{\bar{\tau}_B^*}\right)\bar{\kappa}_B\right], \quad \text{(A3)}$$

where $\tau_i^* = \tau_i^{-1}$.

Prediction 3: Latencies From a Condition

Let

$$\kappa_i = b_1\, \tau_i + b_0 + e_i \quad \text{(A4)}$$

be defined using simple linear regression. Then it can be shown that the rate–amount model (see Equation A1) predicts that, for a given experimental condition (i.e., holding the subscript j constant), the expected value of the mean latency for each individual is predicted to be linearly related to the expected value of each individual's overall mean:

$$E[L_{ij}] = \left(\frac{\delta_j + b_1}{\bar{\delta}_. + b_1}\right) E[L_{i.}] + \left[b_0 - \left(\frac{\delta_j + b_1}{\bar{\delta}_. + b_1}\right) b_0\right] + \left[e_i - \left(\frac{\delta_j + b_1}{\bar{\delta}_. + b_1}\right) e_i\right]. \quad \text{(A5)}$$

The term on the far right of Equation A5 will sum to zero because of properties of linear regression (see Equation A4) and can clearly be seen as an error term for an equation with a linear form.

Prediction 4: Individual Standard Deviations and Overall Means

First, we define an approximation for the expected value of the standard deviations for each individual across conditions, given that no known exact solution exists (see Equation B10).

$$E[S_{L_{i.}}] \approx S_{\mu_{i.}} + a_i. \quad \text{(A6)}$$

For each individual, the standard deviation across experimental conditions is predicted to be linearly related to that individual's overall mean:

$$E[S_{L_{i.}}] \approx \left(\frac{S_{\delta.}}{\bar{\delta}_. + b_1}\right) E[\bar{L}_{i.}] - \left(\frac{S_{\delta.} b_0}{\bar{\delta}_. + b_1}\right) - \left[\left(\frac{S_{\delta.} e_i}{\bar{\delta}_. + b_1}\right) + a_i\right], \quad \text{(A7)}$$

where b_0 and b_1 are simple linear regression parameters obtained from Equation A4 and e_i is the error term from this regression. Although the error terms in Equation A7 (i.e., the values contained in the rightmost square bracket) do have a potentially nonlinear component because of the inclusion of a_i (see Equation B10), the remainder of Equation A7 takes a general linear form, with multiplicative and additive constants.

Prediction 5: Condition Standard Deviations and Group Means

First, we define an approximation for the expected value of the standard deviations for each condition across individuals, given that no known exact solution exists (see Equation B10):

$$E[S_{L_j}] \approx S_{\mu_j} + a_j. \tag{A8}$$

For each experimental condition, the standard deviation across individuals is predicted to be linearly related to that condition's overall mean:

$$E[S_{L_j}] = \left(\frac{S_{\tau^*}}{\bar{\tau}^*}\right) E[\bar{L}_j] + \left(\frac{S_{\tau^*} b_0}{\bar{\tau}^*}\right) + (c_j + a_j), \tag{A9}$$

where $\tau_i^* = \tau_i^{-1}$ and where

$$c_j = \left[\left(\sqrt{\frac{S_{e^*}^2}{(\delta_j + b_1)^2 S_{\tau^*}^2 + 1}}\right) - 1\right](\delta_j + b_1) S_{\tau^*}.$$

Thus, we see that the left and middle portions of Equation A9 compose a linear relationship between the expected value of the sample standard deviation for a condition across individuals and the expected value of the mean for a condition (i.e., including both a multiplicative and additive constants). We also see that the far-right portion contains error terms with potentially nonlinear components. However, these error terms will be roughly linear under a wide range of conditions.

Appendix B

Approximation of the Expected Value of the Sample Standard Deviation for a Set of Heterogeneous Normal Random Variables

Assume $X_i \ldots X_n$ are mutually independent and normally distributed with $N(\mu_i, \sigma_i)$. We are interested in the expected value of the sample standard deviation (S) taken across this set of variables. However, because the squared deviations in the sums of squares used to compute S,

$$B = \sum_{i=1}^{n} (X_i - \bar{X})^2, \tag{B1}$$

are not mutually independent, we define the transformation (Helmert's transformation), which does yield mutually independent transformed variables:

$$Y_i = [i(i+1)]^{-1/2} \left(\sum_{j=1}^{i} X_j - i X_{i+1}\right). \tag{B2}$$

A well-known property of Helmert's transformation is that the sum of the transformed variable Y_i is equal to the sums of squares of the original variables:

$$B = \sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=1}^{n-1} Y_i^2 = \sum_{i=1}^{n-1} \sigma_{Y_i}^2 \chi'^2_1\left(\frac{\mu_{Y_i}^2}{\sigma_{Y_i}^2}\right). \tag{B3}$$

Because Y_i is normal and mutually independent with unique variance and mean, each Y_i^2 is distributed as a weighted noncentral chi-square variable with 1 *df.* The mean and variance of B are therefore:

$$E[B] = \sum_{i=1}^{n-1} \sigma_{Y_i}^2\left(1 + \frac{\mu_{Y_i}^2}{\sigma_{Y_i}^2}\right) = (n-1)\bar{\sigma}^2 + \sum_{i=1}^{n} (\mu_i - \bar{\mu})^2 \tag{B4}$$

and

$$\mathrm{var}(B) = \sum_{i=1}^{n-1} 2(\sigma_{Y_i}^2)^2 \left(1 + 2\frac{\mu_{Y_i}^2}{\sigma_{Y_i}^2}\right). \tag{B5}$$

The distribution of B is unknown but can be effectively approximated by a weighted chi-square variable with the same mean and variance as B (e.g., Kulkarni & Shah, 1995). Thus, we define $c = v_1 \chi_{v_2}^2$, where C is approximately distributed as B, $E(C) = v_1 V_2$, and $\mathrm{var}(C) = V_1^2(2v_2)$.

We then set $E[B] = E[C]$ and $\mathrm{var}(B) = \mathrm{var}(C)$ and solve for v_1 and v_2. We find that $v_1 = \dfrac{\mathrm{var}(B)}{2E[B]}$ and $v_2 = \dfrac{2(E[B])^2}{\mathrm{var}(B)}$.
We now can show that

$$S = \sqrt{B/(n-1)} = \sqrt{v_1/(n-1)}\,\chi_{v_2}. \tag{B6}$$

It is well known that

$$E[\chi_v] = \sqrt{2}\,\frac{\Gamma([v+1]/2)}{\Gamma(v/2)} \approx \frac{4v^{3/2}}{4v+1} \tag{B7}$$

and therefore that

$$E[S] = \sqrt{v_1/(n-1)}\,E[\chi_{v_2}] \approx \sqrt{\mu_B/(n-1)}\,\frac{4v_2}{4v_2+1}. \tag{B8}$$

We can now express $E[S]$ in terms of the expected value and variance of X_i. We can also simplify our answer by recognizing that the ratio involving v_2 above will be very near to one when v_2 is even modestly larger:

$$E[S] \approx \sqrt{\frac{\bar{\sigma}^2 + \sum_{i=1}^{n} (\mu_i - \bar{\mu})^2}{(n-1)}} = \sqrt{\bar{\sigma}^2 + S_\mu^2}. \tag{B9}$$

Finally, we define an arbitrary constant and remove the radical:

$$E[S] \approx S_\mu^2 + a, \tag{B10}$$

where

$$a = \left[\left(\sqrt{\frac{\bar{\sigma}^2}{S_\mu^2 + 1}}\right) - 1\right] S_\mu.$$

We now have an approximation for the expected value of the sample standard deviation, which has both a linear and nonlinear component. Although it can be shown that the nonlinear component a in Equation B10 is approximately linear under a wide range of conditions, we consider this component to be an error term in the development of the rate–amount model presented in Appendix A.

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